Consensus on Moving Neighborhood Model of Peterson Graph[1]

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Abstract

In this paper, we study the consensus problem of multiple agents on a kind of famous graph, Peterson graph. It is an undirected graph with 10 vertices and 15 edges. Each agent randomly walks on this graph and communicates with each other if and only if they coincide on a node at the same time. We conduct numerical study on the consensus problem in this framework and show that global consensus can be achieved.

Keywords: consensus problem; discrete-time protocol; Peterson graph.

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1 Introduction

In a consensus problem, a set of agents aim to make an agreement on some quantities of interest via distributive decision making. The information interactions are based upon local neighboring structure. A consensus is said to be achieved if all agents in the system tend to agree on the quantities of interest as time approaches infinity, cf. survey papers [7, 10] and references therein. Especially, the speed of synchronization is investigated in [3].

Ali Jadbabaie et al. [5] studied a simple model of flocking introduced by Vicsek et al. [14] showing that all agents will reach consensus as time goes on, provided the communication graph switching deterministically over time is periodically jointly connected. Some researchers have also treated random situations, see e.g. [4, 8]. Recently, a new model called moving neighborhood network is introduced in [12]. In this model, each agent carries an oscillator and diffuses in the environment. The computer simulation shows that synchronization is possible even when the communication network is spatially disconnected in general at any given time instant. Subsequently, several researchers have derived analytical results on the moving neighborhood networks, see e.g. [13, 9, 6, 11, 2].

The aim of this paper is to implement consensus on moving neighborhood network modeled by the famous Peterson graph [1]. See Fig. 1. There are many interesting characteristics of Peterson graph in mathematics. For example, it is traceable but not Hamiltonian. That is, it has a Hamiltonian path but doesn't have a Hamiltonian cycle. It is also the canonical example of a hypohamiltonian graph. In this paper, we show that it is possible to reach consensus on them by using moving neighborhood model.

2 Preliminaries

Let $G = (V, E, W)$ be a weighted graph with vertex set V. E is a set of pairs of elements of V called edges. $W = (w_{ij})$ is the weight matrix, in which $w_{ij} > 0$ if $(i, j) \in E$, and $w_{ij} = 0$ otherwise. Consider n identical agents $\{v_1, v_2, \cdots, v_n\}$ as random walkers on G, moving randomly to a neighbor of

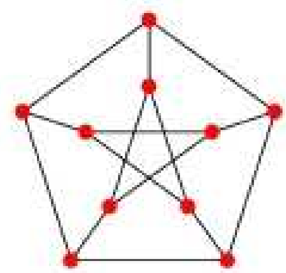

Figure 1: An example of Peterson graph, which has 10 vertices and 15 edges.

their current location in G at any given time. For each agent, the random neighbor that is chosen is not affected by the agent's previous trajectory. The n random walk processes are independent to each other. If v_i and v_j meet at the same node simultaneously, then they can interact with each other by sending information.

Let $X_i(t) \in \mathbb{R}$ be the state of agent v_i at time t. We use the following consensus protocol

$$X_i(t+1) = X_i(t) + \varepsilon \sum_{j \in N_i(t)} b_{ij}(t)(X_j(t) - X_i(t)) \tag{1}$$

where $\varepsilon > 0$ and $N_i(t)$ is the index set of neighbors of agent v_i at time t. The factor $b_{ij} > 0$ for $i \neq j$, and $b_{ii} = 0$ for $1 \leq i \leq n$. Let $A(t) = (a_{ij}(t))$ be the adjacency matrix of the moving neighborhood network, whose entries are given by,

$$a_{ij}(t) = \begin{cases} b_{ij}(t), & (v_i, v_j) \in E(t) \\ 0, & otherwise \end{cases}$$

for $1 \leq i, j \leq n$. Suppose that $\triangle := \max_{1 \leq i \leq n}(\sum_{j=1}^{n} b_{ij}(t))$, and we further assume $\varepsilon \in (0, 1/\triangle)$ for all t. We will show that the states of all agents walking on a Peterson graph reach consensus as time goes on.

3 Numerical examples

For Peterson graph represented in Fig. 1, we take the weight matrix as the adjacency matrix. In addition, we take the $b_{ij}(t)$ randomly from a set of

basic functions such as e^t, $\sin(t)$, $\cos(t)$ and so on. In Fig. 2,3,4,5, we show that the consensus can be achieved asymptotically.

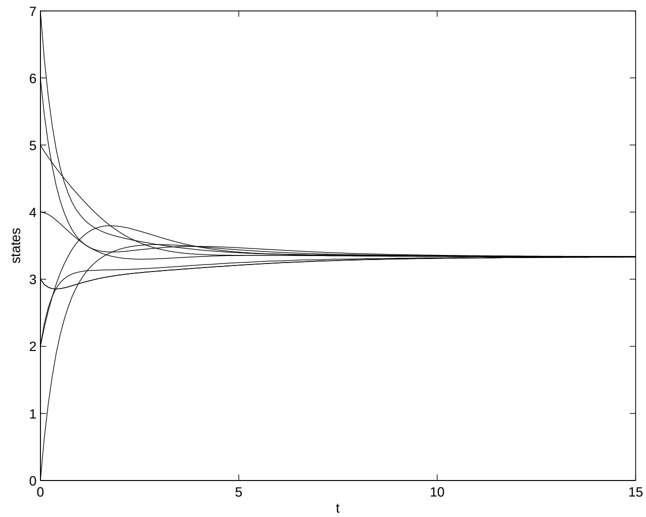

Figure 2: The consensus over moving neighborhood network modeled by a Peterson graph.

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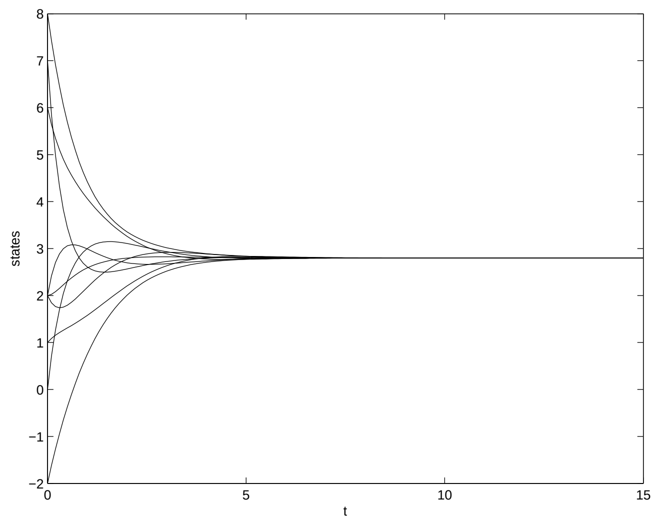

Figure 3: The consensus over moving neighborhood network modeled by a Peterson graph.

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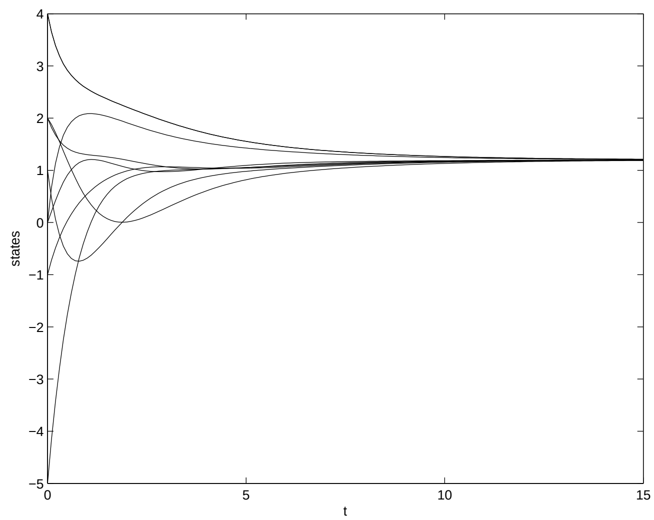

Figure 4: The consensus over moving neighborhood network modeled by a Peterson graph.

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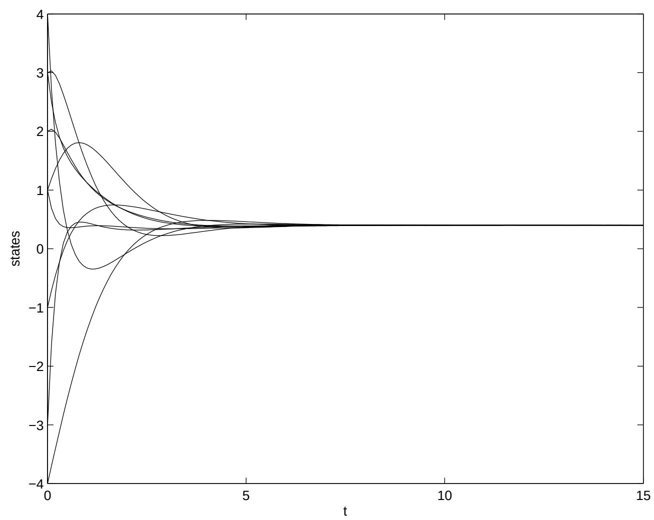

Figure 5: The consensus over moving neighborhood network modeled by a Peterson graph.